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Embedding Flipped $SU(5)$ into $SO(10)$

Chao-Shang Huang,[1] Tianjun Li,[1,2] Chun Liu,[1]
Jonathan P. Shock,[1] Feng Wu,[1] and Yue-Liang Wu[1]

[1] *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, P. R. China*

[2] *Department of Physics and Astronomy,*
Rutgers University, Piscataway, NJ 08854, USA

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Abstract

We embed the flipped $SU(5)$ models into the $SO(10)$ models. After the $SO(10)$ gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetry, we can split the five/one-plets and ten-plets in the spinor **16** and $\overline{\mathbf{16}}$ Higgs fields via the stable sliding singlet mechanism. As in the flipped $SU(5)$ models, these ten-plet Higgs fields can break the flipped $SU(5)$ gauge symmetry down to the Standard Model gauge symmetry. The doublet-triplet splitting problem can be solved naturally by the missing partner mechanism, and the Higgsino-exchange mediated proton decay can be suppressed elegantly. Moreover, we show that there exists one pair of the light Higgs doublets for the electroweak gauge symmetry breaking. Because there exist two pairs of additional vector-like particles with similar intermediate-scale masses, the $SU(5)$ and $U(1)_X$ gauge couplings can be unified at the GUT scale which is reasonably (about one or two orders) higher than the $SU(2)_L \times SU(3)_C$ unification scale. Furthermore, we briefly discuss the simplest $SO(10)$ model with flipped $SU(5)$ embedding, and point out that it can not work without fine-tuning.

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I. INTRODUCTION

The gauge hierarchy problem is one of the main motivations to study the physics beyond the Standard Model (SM). The Higgs boson is needed in the SM to break the electroweak gauge symmetry and give masses to the SM fermions, and the breaking scale is directly related to the Higgs boson mass. However, in quantum field theory, the fermionic masses can be protected against quantum corrections by chiral symmetry, while there is no such symmetry for bosonic masses. The Higgs boson mass (squared) has a quadratic divergence at one loop, and it is unnatural to make a stable weak scale which is hierarchically smaller than the Planck scale. Moreover, an aesthetic motivation for physics beyond the SM is Grand Unified Theories (GUTs) because GUTs can unify all the known gauge interactions, and can give us a simple understanding of the quantum numbers of the SM fermions, etc.

Supersymmetry provides an elegant solution to the gauge hierarchy problem. And the success of gauge coupling unification in the Minimal Supersymmetric Standard Model (MSSM) strongly supports the possibility of supersymmetric GUTs [1, 2]. Other appealing features in supersymmetric GUTs are that the electroweak gauge symmetry is broken by radiative corrections due to the large top quark Yukawa coupling, and that the tiny neutrino masses can be naturally generated by the see-saw mechanism [3]. Therefore, supersymmetric GUTs are promising candidates that can describe all the known fundamental interactions in nature except gravity. However, there are severe problems in the four-dimensional supersymmetric GUTs, especially the doublet-triplet splitting problem and the proton decay problem.

Among the known supersymmetric GUTs, only the flipped $SU(5)$ models can naturally explain the doublet-triplet splitting via a simple and elegant missing partner mechanism [4, 5, 6]. The Higgsino-exchange mediated proton decay problem, which is such a difficulty for the other supersymmetric GUTs, is solved automatically. However, the gauge group of flipped $SU(5)$ models is the product group $SU(5) \times U(1)_X$, not a simple group, so the unifications of the gauge interactions and their couplings are not "grand". As a result, SM fermions in each family do not sit in a single representation of the gauge group, unlike the case in the $SO(10)$ model. In flipped $SU(5)$ models, since the masses of down-type quarks and charged leptons come from different Yukawa couplings, the bottom quark mass is generically not equal to the τ lepton mass at the GUT scale, which is one of the consistent predictions in the other supersymmetric GUTs, e.g., $SU(5)$. The grand unification of the gauge interactions, and the unification of each family of the SM fermions into a single representation can be achieved by embedding the flipped $SU(5)$ into $SO(10)$. However, it is well-known that the missing partner mechanism can not work, because the partners that were missing in the $SU(5) \times U(1)_X$ multiplets are indeed appear in the larger $SO(10)$ multiplets. To solve this problem, two kinds of models were proposed: the five-dimensional orbifold $SO(10)$ models [7], and the four-dimensional $SO(10) \times SO(10)$ models with bi-spinor link Higgs fields [8] (For other $SO(10)$ models with flipped $SU(5)$ embedding, please see Refs. [9].).

In this paper, we would like to embed the flipped $SU(5)$ models into the four-dimensional $SO(10)$ models where the missing partner mechanism can still work elegantly. In the flipped $SU(5)$ models, the Higgs fields H and $\overline{H}$, which break the flipped $SU(5)$ gauge symmetry down to the SM gauge symmetry, are one pair of vector-like fields in the $(\mathbf{10}, \mathbf{1})$ and $(\overline{\mathbf{10}}, -\mathbf{1})$ representations of $SU(5) \times U(1)_X$, respectively. When we embed the flipped $SU(5)$ into $SO(10)$, these Higgs fields H and $\overline{H}$ respectively are embedded into the Higgs fields Σ and $\overline{\Sigma}$ in the spinor $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations of $SO(10)$. The missing partners for the MSSM Higgs doublets H_u and H_d respectively belong to the $(\overline{\mathbf{5}}, -\mathbf{3})$ and $(\mathbf{5}, \mathbf{3})$ of the Σ and $\overline{\Sigma}$ when we decompose the $SO(10)$ spinor representations into the $SU(5) \times U(1)_X$ representations (for detail decompositions please see Appendix A). Also, in the flipped $SU(5)$ models, the Higgs fields $\overline{h}$ and h, which include the Higgs doublets H_u and H_d, are in $(\overline{\mathbf{5}}, \mathbf{2})$ and $(\mathbf{5}, -\mathbf{2})$ representations, respectively. Interestingly, the Higgs fields $\overline{h}$ and h in our models can form a $\mathbf{10}$ representation Higgs field $h_{\mathbf{10}}$ of $SO(10)$. Note that we will break the $SO(10)$ gauge symmetry down to the flipped $SU(5)$ gauge symmetry at the GUT scale M_{GUT}, and further down to the SM gauge symmetry at the $SU(2)_L \times SU(3)_C$ unification scale M_{23}. So, to have the successful missing partner mechanism for the doublet-triplet splitting, we must split the five-plets and ten-plets in the Σ and $\overline{\Sigma}$, *i. e.*, the five-plets in the Σ and $\overline{\Sigma}$ must have mass around the scale M_{GUT} while the corresponding ten-plets should remain massless after the $SO(10)$ gauge symmetry breaking.

We construct the three-family $SO(10)$ models with two adjoint Higgs fields Φ and Φ', Σ, $\overline{\Sigma}$, $h_{\mathbf{10}}$, one pair of spinor $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations χ and $\overline{\chi}$, and several singlets. After the $SO(10)$ gauge symmetry is broken down to the flipped $SU(5)$ gauge symmetry, the five/one-plets and ten-plets in the multiplets $\overline{\chi}$ and Σ, and $\overline{\Sigma}$ and χ can be splitted via the sliding singlet mechanism. And we can show that this sliding singlet mechanism is stable. Similar to the flipped $SU(5)$ models, we can break the gauge symmetry down to the SM gauge symmetry by giving vacuum expectation values (VEVs) to the neutral singlet components of H and $\overline{H}$. The doublet-triplet splitting can be realized by the simple missing partner mechanism, and the Higgsino-exchange mediated proton decay is negligible. Moreover, we show that there exists one pair of the light Higgs doublets mainly from H_u and H_d for the electroweak gauge symmetry breaking. Since there exist two pairs of vector-like particles (mainly from the correspoding components in χ and $\overline{\chi}$) with roughly the same intermediate-scale masses whose SM quantum numbers are $\left((\mathbf{3}, \mathbf{2}, \mathbf{\frac{1}{6}}), (\mathbf{\overline{3}}, \mathbf{2}, -\mathbf{\frac{1}{6}})\right)$ and $\left((\mathbf{\overline{3}}, \mathbf{1}, \mathbf{\frac{1}{3}}) + (\mathbf{3}, \mathbf{1}, -\mathbf{\frac{1}{3}})\right)$, the $SU(5) \times U(1)_X$ gauge coupling unification can be achieved at the GUT scale which is reasonably (about one or two orders) higher than the $SU(2)_L \times SU(3)_C$ unification scale [10, 11]. Therefore, we can keep the beautiful features and get rid of the drawbacks of the flipped $SU(5)$ models in our $SO(10)$ models.

Furthermore, we briefly consider the simplest $SO(10)$ model with flipped $SU(5)$ embedding, and point out that we have to fine-tune some mass parameters so that the model can be consistent. We also explain how to generate the suitable vector-like mass for χ and $\overline{\chi}$.

This paper is organized as follows: in Section II we briefly review the flipped $SU(5)$ models, and the sliding singlet mechanism. We present our $SO(10)$ models in Section III. Moreover, we consider the mixings between the light and superheavy particles, and study gauge coupling unification in Section IV. Our remarks on the simplest $SO(10)$ model and the vector-like mass for χ and $\overline{\chi}$ are given in Section V. Section VI is our discussion and conclusions. We present the $SO(10)$ generators in the spinor representations in Appendix A.

II. BRIEF REVIEW

In this Section, we would like to briefly review the flipped $SU(5)$ models [4, 5], and the sliding singlet mechanism [12].

A. The flipped $SU(5)$ Models

First, let us consider the flipped $SU(5)$ models [4, 5]. We can define the generator $U(1)_{Y'}$ in $SU(5)$ as

$$T_{\mathrm{U(1)_{Y'}}} \equiv \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right), \tag{1}$$

and the hypercharge is given by

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right). \tag{2}$$

There are three families of the SM fermions with the following $SU(5) \times U(1)_X$ quantum numbers

$$F_i = (\mathbf{10}, \mathbf{1}),\ \bar{f}_i = (\bar{\mathbf{5}}, \mathbf{-3}),\ \bar{l}_i = (\mathbf{1}, \mathbf{5}), \tag{3}$$

where $i = 1, 2, 3$. As an example, the particle assignments for the first family are

$$F_1 \ = \ (Q_1,\ D_1^c,\ N_1^c)\ ,\ \ \overline{f}_1 \ = \ (U_1^c,\ L_1)\ ,\ \ \overline{l}_1 \ = \ E_1^c\ , \tag{4}$$

where Q and L are respectively the superfields of the left-handed quark and lepton doublets, U^c, D^c, E^c and N^c are the CP conjugated superfields for the right-handed up-type quark, down-type quark, lepton and neutrino, respectively. In addition, to give heavy masses to the right-handed neutrinos, we add three singlets ϕ_i.

To break the GUT and electroweak gauge symmetries, we introduce two pairs of vector-like Higgs fields

$$H \ = \ (\mathbf{10}, \mathbf{1})\ ,\ \ \overline{H} = (\overline{\mathbf{10}}, \mathbf{-1})\ ,\ \ h = (\mathbf{5}, \mathbf{-2})\ ,\ \ \overline{h} = (\bar{\mathbf{5}}, \mathbf{2})\ . \tag{5}$$

We label the states in the H multiplet by the same symbols as in the F multiplet, and for $\overline{H}$ we just add "bar" above the fields. Explicitly, the Higgs particles are

$$H \ = \ (Q_H,\ D_H^c,\ N_H^c)\ ,\ \ \overline{H} \ = \ (\overline{Q}_{\overline{H}},\ \overline{D}^c_{\overline{H}},\ \overline{N}^c_{\overline{H}})\ , \tag{6}$$

$$h \ = \ (D_h,\ D_h,\ D_h,\ H_d)\ ,\ \ \overline{h} \ = \ (\overline{D}_{\overline{h}},\ \overline{D}_{\overline{h}}, \overline{D}_{\overline{h}},\ H_u)\ , \tag{7}$$

where H_d and H_u are the two Higgs doublets in the MSSM.

To break the $SU(5) \times U(1)_X$ gauge symmetry down to the SM gauge symmetry, we introduce the following superpotential

$$W = \lambda_1 HHh + \lambda_2 \overline{H}\overline{H}\overline{h} + S(\overline{H}H - M_H^2) , \tag{8}$$

where S is a singlet, and λ_1 and λ_2 are Yukawa couplings. There is only one F-flat and D-flat direction, which can always be rotated along the N_H^c and $\overline{N_{\overline{H}}^c}$ directions. So, we obtain that $\langle N_H^c \rangle = \langle \overline{N_{\overline{H}}^c} \rangle = M_H$. In addition, the superfields H and $\overline{H}$ are eaten and acquire large masses via the Higgs mechanism with supersymmetry, except for D_H^c and $\overline{D_{\overline{H}}^c}$. The superpotential terms $\lambda_1 HHh$ and $\lambda_2 \overline{H}\overline{H}\overline{h}$ combine the D_H^c and $\overline{D_{\overline{H}}^c}$ with the D_h and $\overline{D_{\overline{h}}}$, respectively, to form the massive eigenstates with masses $2\lambda_1 \langle N_H^c \rangle$ and $2\lambda_2 \langle \overline{N_{\overline{H}}^c} \rangle$. Since there are no partners in H and $\overline{H}$ for H_u and H_d, we naturally obtain the doublet-triplet splitting due to the missing partner mechanism. Because the triplets in h and $\overline{h}$ only have small mixing through the μ term, the Higgsino-exchange mediated proton decay are negligible, *i.e.*, we do not have the dimension-5 proton decay problem.

The SM fermion masses are from the following superpotential

$$W_{\text{Yukawa}} = \frac{1}{2} y_{ij}^D F_i F_j h + y_{ij}^{U\nu} F_i \overline{f}_j \overline{h} + y_{ij}^E \overline{l}_i \overline{f}_j h + \mu h \overline{h} + y_{ij}^N \phi_i \overline{H} F_j , \tag{9}$$

where y_{ij}^D, $y_{ij}^{U\nu}$, y_{ij}^E and y_{ij}^N are Yukawa couplings, and μ is the bilinear Higgs mass term.

After the $SU(5) \times U(1)_X$ gauge symmetry is broken down to the SM gauge symmetry, the above superpotential gives

$$\begin{aligned} W_{SSM} \;=\; & y_{ij}^D D_i^c Q_j H_d + y_{ji}^{U\nu} U_i^c Q_j H_u + y_{ij}^E E_i^c L_j H_d + y_{ij}^{U\nu} N_i^c L_j H_u \\ & + \mu H_d H_u + y_{ij}^N \langle \overline{N_{\overline{H}}^c} \rangle \phi_i N_j^c + \cdots (\text{decoupled below } M_{GUT}). \end{aligned} \tag{10}$$

B. Sliding Singlet Mechanism

The sliding singlet mechanism was originally proposed in the supersymmetric $SU(5)$ model [12], where the Higgs superpotential is

$$W = W(\Phi) + \overline{H}_{\bar{\mathbf{5}}} (\Phi + S) H_{\mathbf{5}} , \tag{11}$$

where Φ is an $SU(5)$ adjoint Higgs field, S is a SM singlet, and $\overline{H}_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ are the anti-fundamental and fundamental Higgs fields which respectively contain one pair of Higgs doublets H_d and H_u.

With suitable superpotential $W(\Phi)$ for Φ, one assumes that Φ obtains the following VEV

$$\Phi \;=\; \mathrm{diag}\left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2}\right) V_{\Phi} \ . \tag{12}$$

Then, the $SU(5)$ gauge symmetry is broken down to the SM gauge symmetry.

The F-flatness conditions for the F-terms of $\overline{H}_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$, which is valid at a supersymmetric minimum, give the following equations

$$\left(\langle\Phi\rangle+\langle S\rangle\right)\langle H_{\mathbf{5}}\rangle=0\ ,\ \ \langle\overline{H}_{\bar{\mathbf{5}}}\rangle\left(\langle\Phi\rangle+\langle S\rangle\right)=0\ . \tag{13}$$

To break the electroweak gauge symmetry, the Higgs doublets H_d and H_u are supposed to obtain VEVs around the electroweak scale, From F-flatness conditions $F_{H_d}=F_{H_u}=0$, we obtain

$$\langle S\rangle \;=\; -\frac{1}{2}V_{\Phi} \ . \tag{14}$$

Therefore, we have

$$\langle\Phi\rangle+\langle S\rangle \;=\; \mathrm{diag}\left(-\frac{5}{6},-\frac{5}{6},-\frac{5}{6},0,0\right) V_{\Phi} \ . \tag{15}$$

As a result, the color triplets in $\overline{H}_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ will obtain vector-like mass around V_{Φ}, while the doublets will remain massless after the $SU(5)$ gauge symmetry breaking. Because the singlet slides to cancel off the VEV of the adjoint Higgs field in the $SU(2)_L$ block, this mechanism is called the sliding singlet mechanism.

However, the sliding singlet mechanism for supersymmetric $SU(5)$ model breaks down due to the supersymmetry breaking [13]. The potential from the F-terms of $\overline{H}_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ only gives the electroweak-scale mass $(\sqrt{(\langle H_d^0\rangle)^2+(\langle H_u^0\rangle)^2})$ to S, and the soft supersymmetry breaking gives S mass around the supersymmetry breaking scale M_S. However, S couples to the triplets in $\overline{H}_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ with masses around the GUT scale, so, the one-loop tadpole graphs with the triplets running around the loop induce the following two terms in the potential in the low energy effective theory that destroy the above doublet-triplet splitting

$$T_1 \;=\; \mathcal{O}(m_g^2 M_{GUT})S+\ \mathrm{H.C.}\ ,\ \ T_2 \;=\; \mathcal{O}(m_g M_{GUT})F_S+\ \mathrm{H.C.}\ , \tag{16}$$

where m_g is the gravitino mass, which is usually around M_S.

The T_1 term will shift the VEV of S from its supersymmetric minimum $-V_{\Phi}/2$ by the following amount

$$\delta\langle S\rangle \;\sim\; \frac{\mathcal{O}(m_g^2 M_{GUT})}{\mathcal{O}(M_S^2)+(\langle H_d^0\rangle)^2+(\langle H_u^0\rangle)^2} \;\sim\; \mathcal{O}(M_{GUT})\ , \tag{17}$$

and then the doublets in $\overline{H}_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ will obtain the vector-like mass around the GUT scale.

In addition, after we integrate out the auxiliary field F_S, the T_2 term gives the following term in the potential

$$V \supset |\overline{H}_{\bar{\mathbf{5}}} H_{\mathbf{5}} + \mathcal{O}(m_g M_{GUT})|^2 . \tag{18}$$

Thus, the VEVs of H_d^0 and H_u^0 are around the scale $\sqrt{m_g M_{GUT}}$, which is inconsistent with the known value of $\sqrt{(\langle H_d^0 \rangle)^2 + (\langle H_u^0 \rangle)^2} \simeq 246.2$ GeV.

In the gauge mediated supersymmetry breaking scenario, the gravitino mass can be very light and below the keV scale. However, the sliding singlet mechanism still may not work [14].

The sliding singlet mechanism can be successfully applied to the rank five or higher GUT groups [15, 16, 17], for example, the $SU(6)$ and E_6 models, etc. The point is that the corresponding Higgs fields like the $\overline{H}_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ in the $SU(5)$ model can have the very large or GUT-scale VEVs. Let us briefly comment on the $SU(6)$ models. To keep the F-flatness and have one pair of light Higgs doublets, we need at least three pairs of vector-like particles in the $SU(6)$ fundamental $\mathbf{6}$ and anti-fundamental $\bar{\mathbf{6}}$ representations. In the known model, there are four pairs of such particles [16].

III. $SO(10)$ MODELS

We will construct the $SO(10)$ models where the gauge symmetry is broken down to the flipped $SU(5)$ gauge symmetry by giving VEVs to the adjoint Higgs fields, and further down to the SM gauge symmetry by giving VEVs to the H and $\overline{H}$. We denote the SM fermions as ψ_i which form the spinor $\mathbf{16}$ representation. We introduce two adjoint $\mathbf{45}$ representation Higgs fields Φ and Φ', one pair of the spinor $\mathbf{16}$ and $\overline{\mathbf{16}}$ representation Higgs fields Σ and $\overline{\Sigma}$, one $\mathbf{10}$ representation Higgs field $h_{\mathbf{10}}$, one pair of the spinor $\mathbf{16}$ and $\overline{\mathbf{16}}$ representation vector-like particles χ and $\overline{\chi}$, and nine singlets ϕ_i, S, S', S_i, and S_Σ where $i = 1, 2, 3$. The complete particle content is given in Table I.

In terms of the particles in the flipped $SU(5)$ models, we have

$$\psi_i \ = \ (F_i, \ \bar{f}_i, \ \bar{l}_i) \ ; \quad h_{\mathbf{10}} \ = \ (h, \ \overline{h}) \ . \tag{19}$$

In our convention, for one pairs of the spinor $\mathbf{16}$ and $\overline{\mathbf{16}}$ representation chiral superfields K and $\overline{K}$, we denote their components like the SM fermions as following

$$K \ = \ (K_F, \ K_{\bar{f}}, \ K_{\bar{l}}) \ , \quad \overline{K} \ = \ (\overline{K}_{\overline{F}}, \ \overline{K}_f, \ \overline{K}_l) \ , \tag{20}$$

TABLE I: Particle content in $SO(10)$ models.

Representation	Chiral Superfields
45	$\Phi;\quad \Phi'$
16	$\psi_i;\quad \Sigma;\quad \chi$
$\overline{\mathbf{16}}$	$\overline{\Sigma};\quad \overline{\chi}$
10	$h_{\mathbf{10}}$
1	$\phi_i;\quad S;\quad S';\quad S_i;\quad S_\Sigma$

where

$$
\begin{aligned}
K_F &= (Q_K,\ D^c_K,\ N^c_K)\ ,\quad K_{\bar f} = (U^c_K,\ L_K)\ ,\\
\overline{K}_{\overline{F}} &= (\overline{Q}_{\overline{K}}, \overline{D}^c_{\overline{K}}, \overline{N}^c_{\overline{K}})\ ,\quad \overline{K}_f = (\overline{U}^c_{\overline{K}},\ \overline{L}_{\overline{K}})\ .
\end{aligned}
\tag{21}
$$

The only exception is that similar to the flipped $SU(5)$ models, we denote the Higgs fields Σ_F and $\overline{\Sigma}_{\overline{F}}$ as H and $\overline{H}$, respectively. To be concrete, we have

$$
\Sigma \ = \ (H,\ \Sigma_{\bar f},\ \Sigma_{\bar l})\ ,\quad \overline{\Sigma} \ = \ (\overline{H},\ \overline{\Sigma}_f,\ \overline{\Sigma}_l)\ .
\tag{22}
$$

The superpotential is

$$
\begin{aligned}
W \ &= \ W(\Phi,\Phi') + W(\Sigma,\overline{\Sigma}) + y_{ij}\psi_i h_{\mathbf{10}}\psi_j + y^N_{ij}\phi\overline{\Sigma}\psi_j + \frac{1}{2}\mu h_{\mathbf{10}} h_{\mathbf{10}} + \lambda_1 \Sigma h_{\mathbf{10}}\Sigma \\
&\quad + \lambda_2 \overline{\Sigma} h_{\mathbf{10}}\overline{\Sigma} + \lambda_3\overline{\chi}(\Phi + \lambda_4 S)\Sigma + \lambda_5\overline{\Sigma}(\Phi' + \lambda_6 S')\chi + M_\chi\overline{\chi}\chi\ ,
\end{aligned}
\tag{23}
$$

where y_{ij}, y^N_{ij}, and λ_i ($i = 1, 2, ..., 6$) are Yukawa couplings, and μ and M_χ are vector-like masses. The general superpotential $W(\Phi,\Phi')$ for Φ and Φ', and the simple superpotential $W(\Sigma,\overline{\Sigma})$ for Σ and $\overline{\Sigma}$ are

$$
\begin{aligned}
W(\Phi,\Phi') \ &= \ \kappa\Phi^3 + M_\Phi\Phi^2 + \lambda_7 S_1(\Phi^2 - m^2_{11}) + \kappa'\Phi'^3 + M_{\Phi'}\Phi'^2 + \lambda'_7 S_2(\Phi'^2 - m^2_{22}) \\
&\quad + M_{\Phi\Phi'}\Phi\Phi' + \lambda_8 S_3(\Phi\Phi' - m^2_{12})\ ,
\end{aligned}
\tag{24}
$$

$$
W(\Sigma,\overline{\Sigma}) \ = \ S^2_\Sigma(\Sigma\overline{\Sigma} - M^2_H)\ ,
\tag{25}
$$

where κ, κ', λ_7, λ'_7, and λ_8 are Yukawa couplings, and M_Φ, $M_{\Phi'}$, $M_{\Phi\Phi'}$, m_{11}, m_{22}, m_{12}, and M_H are mass parameters.

Let us briefly comment on $W(\Phi, \Phi')$. First, we must have at least one term which couples Φ and Φ' so that we only have one global $SO(10)$ symmetry in $W(\Phi, \Phi')$, *i. e.*, the $SO(10)$ gauge symmetry. Otherwise, we will have some unwanted massless Nambu-Goldstone bosons. Second, some of the Yukawa couplings and mass parameters in $W(\Phi, \Phi')$ should be zero. For example, m_{11}, m_{22}, and m_{12} can not be all non-zero in general, otherwise, we need to fine-tune these masses to satisfy the F-flatness conditions $F_{S_i} = 0$. Let us present a simple $W(\Phi, \Phi')$

$$W(\Phi, \Phi') = M_{\Phi\Phi'}\Phi\Phi' + \lambda_8 S_3(\Phi\Phi' - m_{12}^2) \ . \tag{26}$$

The flatness of F-term of S_3 ($F_{S_3} = 0$) implies that $\langle\Phi\rangle \neq 0$ and $\langle\Phi'\rangle \neq 0$. Also, the F-flatnesses of the F-terms of Φ and Φ' ($F_\Phi = F_{\Phi'} = 0$) imply that $\langle\Phi\rangle = \langle\Phi'\rangle \neq 0$ and $\langle S_3\rangle \neq 0$. By the way, at very high temperature, the $SO(10)$ gauge symmetry will be restored when we consider the superpotential at finite temperature.

The gauge fields of $SO(10)$ are in the adjoint representation of $SO(10)$ with dimension $\mathbf{45}$. Under the gauge group $SU(5) \times U(1)_X$, the $SO(10)$ gauge fields decompose as [18]

$$\mathbf{45} = (\mathbf{24}, \mathbf{0}) \oplus (\mathbf{10}, -\mathbf{4}) \oplus (\overline{\mathbf{10}}, \mathbf{4}) \oplus (\mathbf{1}, \mathbf{0}) \ . \tag{27}$$

To break the $SO(10)$ gauge symmetry down to the flipped $SU(5)$ gauge symmetry via adjoint Higgs fields, we need to give the VEVs to their singlet components.

As we explained in the Introduction, to achieve the doublet-triplet splitting via the missing partner mechanism, we must split the five/one-plets and ten-plets in the Σ and $\overline{\Sigma}$ during the $SO(10)$ gauge symmetry breaking. In order to give the GUT-scale masses to the $\Sigma_{\bar{f}}$, $\Sigma_{\bar{l}}$, $\overline{\Sigma}_f$ and $\overline{\Sigma}_l$ while keep H and $\overline{H}$ massless when we break the $SO(10)$ gauge symmetry down to the flipped $SU(5)$ gauge symmetry, we should express the $SO(10)$ generators in the spinor representations which are 16×16 matries and are given in Appendix A. Note that when the $U(1)_X$ generator $T_{U(1)_X}$ acts on the spinor representation $\mathbf{16}$, it gives us the corresponding $U(1)_X$ charges of the particles belong to $\mathbf{16}$. So, we obtain the generator for $U(1)_X$

$$T_{U(1)_X} \ = \ \mathrm{diag}(1, 1, 1, -3, 1, 1, 1, -3, 1, 1, 1, 5, -3, -3, -3, 1) \ . \tag{28}$$

For simplicity, we assume that the Φ and Φ' obtain the VEVs at the GUT scale due to the superpotential $W(\Phi, \Phi')$, and the F-flatness conditions for the F-terms of Φ, Φ' and S_i are satisfied by choosing suitable Yukawa couplings and mass parameters in $W(\Phi, \Phi')$. The explicit VEVs for Φ and Φ' are

$$\begin{aligned} \langle \Phi \rangle &= \mathrm{diag}(1,1,1,-3,1,1,1,-3,1,1,1,5,-3,-3,-3,1)\, V_{\Phi} \ , \\ \langle \Phi' \rangle &= \mathrm{diag}(1,1,1,-3,1,1,1,-3,1,1,1,5,-3,-3,-3,1)\, V_{\Phi'} \ , \end{aligned} \tag{29}$$

where V_{Φ} and $V_{\Phi'}$ are around the GUT scale.

The F-flatness conditions for the F-terms of $\overline{\chi}$ and χ, which is valid at a supersymmetric minimum, give the following equations

$$(\langle \Phi \rangle + \lambda_4 \langle S \rangle) \langle \Sigma \rangle = 0 \ , \ \langle \overline{\Sigma} \rangle (\langle \Phi' \rangle + \lambda_6 \langle S' \rangle) = 0 \ . \tag{30}$$

To break the flipped $SU(5)$ gauge symmetry down to the SM gauge symmetry, we give VEVs to $N^c_H \subset H \subset \Sigma$ and $\overline{N^c_H} \subset \overline{H} \subset \overline{\Sigma}$ at the $SU(3)_C \times SU(2)_L$ unification scale M_{23}, which is around 3.7×10^{16} GeV. From the F-flatness conditions $F_{N^c_H} = F_{\overline{N^c_H}} = 0$, we obtain

$$\langle S \rangle \ = \ -\frac{V_{\Phi}}{\lambda_4} \ , \ \langle S' \rangle \ = \ -\frac{V_{\Phi'}}{\lambda_6} \ . \tag{31}$$

Thus, we have

$$\begin{aligned} \langle \Phi \rangle + \lambda_4 \langle S \rangle &= \mathrm{diag}(0,0,0,-4,0,0,0,-4,0,0,0,4,-4,-4,-4,0)\, V_{\Phi} \ , \\ \langle \Phi' \rangle + \lambda_6 \langle S' \rangle &= \mathrm{diag}(0,0,0,-4,0,0,0,-4,0,0,0,4,-4,-4,-4,0)\, V_{\Phi'} \ . \end{aligned} \tag{32}$$

Then we have the following vector-like mass terms for the pairs $(\overline{\chi}_f, \Sigma_{\overline{f}})$, $(\overline{\chi}_l, \Sigma_{\overline{l}})$, $(\overline{\Sigma}_f, \chi_{\overline{f}})$, and $(\overline{\Sigma}_l, \chi_{\overline{l}})$

$$V \ \supset \ -4\lambda_3 V_{\Phi} \left(\overline{\chi}_f \Sigma_{\overline{f}} - \overline{\chi}_l \Sigma_{\overline{l}} \right) - 4\lambda_5 V_{\Phi'} \left(\overline{\Sigma}_f \chi_{\overline{f}} - \overline{\Sigma}_l \chi_{\overline{l}} \right) \ , \tag{33}$$

where for simplicity we neglect the M_{χ}, which will be shown to be very small compared to the scales M_{GUT} and M_{23} so that we can have one pair of the light Higgs doublets for the electroweak gauge symmetry breaking. However, the particles $\overline{\chi}_{\overline{F}}$, H, $\overline{H}$ and χ_F are massless if we neglect M_{χ}. Thus, we split the five/one-plets and ten-plets in the multiplets $\overline{\chi}$ and Σ, and $\overline{\Sigma}$ and χ via the sliding singlet mechanism after we break the $SO(10)$ gauge symmetry down to the flipped $SU(5)$ gauge symmetry.

As discussed in the brief review of the flipped $SU(5)$ models, we break the $SU(5) \times U(1)_X$ gauge symmetry down to the SM gauge symmetry by giving VEVs to the N^c_H and $\overline{N}^c_{\overline{H}}$ of H and $\overline{H}$. The superfields H and $\overline{H}$ are eaten and acquire large masses via the Higgs mechanism with supersymmetry, except for D^c_H and $\overline{D}^c_{\overline{H}}$. And the superpotential $\lambda_1 HHh \subset \lambda_1 \Sigma h_{\mathbf{10}} \Sigma$ and $\lambda_2 \overline{HHh} \subset \lambda_2 \overline{\Sigma} h_{\mathbf{10}} \overline{\Sigma}$ combine the D^c_H and $\overline{D}^c_{\overline{H}}$ with the D_h and $\overline{D}_{\overline{h}}$, respectively, to form the massive eigenstates with masses $2\lambda_1 \langle N^c_H \rangle$ and $2\lambda_2 \langle \overline{N}^c_{\overline{H}} \rangle$. So, we solve the doublet-triplet splitting problem naturally via the missing partner mechanism. Because the triplets in h and $\overline{h}$ of $h_{\mathbf{10}}$ only have small mixing through the μ term, the Higgsino-exchange mediated proton decay are negligible, *i. e.*, we do not have the dimension-5 proton decay problem.

Let us show that our sliding singlet mechanism is stable. The T_1 type tadpoles will shift the VEVs of S and S' from its supersymmetric minimum by the following amount

$$\delta \langle S \rangle ~\sim~ \frac{\mathcal{O}(m_g^2 M_{GUT})}{\lambda_3^2 \lambda_4^2 (\langle N^c_H \rangle)^2} ~,~ \delta \langle S' \rangle ~\sim~ \frac{\mathcal{O}(m_g^2 M_{GUT})}{\lambda_5^2 \lambda_6^2 (\langle \overline{N}^c_{\overline{H}} \rangle)^2} ~. \tag{34}$$

It is obvious that these shifting effects are tiny and can be neglected.

Moreover, after we integrate out the auxiliary fields F_S and $F_{S'}$, the T_2 type tadpoles will give us the following terms in the potential

$$V ~\supset~ |\lambda_3 \lambda_4 \overline{\chi} \Sigma + \mathcal{O}(m_g M_{GUT})|^2 + |\lambda_5 \lambda_6 \overline{\Sigma} \chi + \mathcal{O}(m_g M_{GUT})|^2 ~. \tag{35}$$

Then, we obtain

$$\langle \overline{N}^c_{\overline{\chi}} \rangle ~\sim~ -\frac{\mathcal{O}(m_g M_{GUT})}{\lambda_3 \lambda_4 \langle N^c_H \rangle} ~,~ \langle N^c_{\chi} \rangle ~\sim~ -\frac{\mathcal{O}(m_g M_{GUT})}{\lambda_5 \lambda_6 \langle \overline{N}^c_{\overline{H}} \rangle} ~. \tag{36}$$

Because Σ and $\overline{\Sigma}$, or χ and $\overline{\chi}$ do not contain the one pair of Higgs doublets H_d and H_u in the MSSM, it is fine that we have very small non-zero VEVs for N^c_χ and $\overline{N}^c_{\overline{\chi}}$ compared to the scales M_{GUT} and M_{23}.

Moreover, from the F-flatness conditions for the F-terms of $\overline{\chi}$ and χ, we obtain

$$\langle \Phi \rangle + \lambda_4 \langle S \rangle ~\sim~ \frac{\mathcal{O}(m_g M_{GUT})}{\lambda_3 \lambda_5 \lambda_6 \langle \overline{N}^c_{\overline{H}} \rangle \langle N^c_H \rangle} ~M_\chi ~,~~ \langle \Phi' \rangle + \lambda_6 \langle S' \rangle ~\sim~ \frac{\mathcal{O}(m_g M_{GUT})}{\lambda_3 \lambda_4 \lambda_5 \langle \overline{N}^c_{\overline{H}} \rangle \langle N^c_H \rangle} ~M_\chi ~. \tag{37}$$

So, the variations on $\langle \Phi \rangle + \lambda_4 \langle S \rangle$ and $\langle \Phi' \rangle + \lambda_6 \langle S' \rangle$ are also very small compared to the scales M_{GUT} and M_{23}, and will not affect the splittings of the five/one-plets and ten-plets in the multiplets $\overline{\chi}$ and Σ, and $\overline{\Sigma}$ and χ. Especially, for the gauge mediated supersymmetry breaking, the gravitino mass can be around the keV scale, and these variations are completely negligible. Therefore, our sliding singlet mechanism is stable. By the way, the VEVs of Φ, S, Φ', S', N^c_H, and $\overline{N}^c_{\overline{H}}$ will be shifted by tiny amount due to non-zero $\langle N^c_\chi \rangle$ and $\langle \overline{N}^c_{\overline{\chi}} \rangle$.

In the following discussions, for simplicity we will neglect the VEVs of N^c_χ and $\overline{N}^c_{\overline{\chi}}$ that are very small compared to the V_Φ, $V_{\Phi'}$, $\langle N^c_H \rangle$, and $\langle \overline{N}^c_{\overline{H}} \rangle$.

IV. PHENOMENOLOGICAL CONSEQUENCES

In this Section, we will study the mixings between the light and superheavy particles, and the gauge coupling unification.

A. Light and Superheavy Particle Mixings

After the flipped $SU(5)$ gauge symmetry breaking, the possible light particles are three families of the SM fermions, one pair of the Higgs doublets H_d and H_u, and one pair of the $\mathbf{10}$ representation χ_F and $\overline{\mathbf{10}}$ representation $\overline{\chi_F}$ in χ and $\overline{\chi}$. However, to make sure that H_d, H_u, χ_F, and $\overline{\chi_F}$ are indeed light, we must calculate all the possible mixing mass matrices between these particles and superheavy particles. There are three types of relevant particle mixings:

(1) In the $SU(5)$ language, the doublets (X, Y)- and $(\overline{X}, \overline{Y})$-type particles in the $(\mathbf{24}, \mathbf{0})$ decomposed representations of the Φ and Φ' have the same SM quantum numbers as the quark doublet and its Hermitian conjugate. After N^c_H and $\overline{N}^c_{\overline{H}}$ obtain VEVs, they will mix with the Q_χ and $\overline{Q}_{\overline{\chi}}$ in χ_F and $\overline{\chi_F}$. Let us denote the (X, Y)- and $(\overline{X}, \overline{Y})$-type particles in Φ as Q_Φ and $\overline{Q}_\Phi$, and in Φ' as $Q_{\Phi'}$ and $\overline{Q}_{\Phi'}$. The mass terms in the superpotential are

$$
\begin{aligned}
W \supset\ & M^{11}_{XY} \overline{Q}_\Phi Q_\Phi + M^{12}_{XY} \overline{Q}_\Phi Q_{\Phi'} + M^{21}_{XY} \overline{Q}_{\Phi'} Q_\Phi + M^{22}_{XY} \overline{Q}_{\Phi'} Q_{\Phi'} \\
& + \lambda_3 \langle N^c_H \rangle \overline{Q}_{\overline{\chi}} Q_\Phi + \lambda_5 \langle \overline{N}^c_{\overline{H}} \rangle \overline{Q}_{\Phi'} Q_\chi + M_\chi \overline{Q}_{\overline{\chi}} Q_\chi \ ,
\end{aligned}
\tag{38}
$$

where M^{ij}_{XY} are the mass parameters around the GUT scale. The corresponding mass matrix for the basis $(\overline{Q}_\Phi, \overline{Q}_{\Phi'}, \overline{Q}_{\overline{\chi}})^t$ versus $(Q_\Phi, Q_{\Phi'}, Q_\chi)$, where t is transpose, are the following

$$
M_{XYQ\overline{Q}} = \begin{pmatrix} M^{11}_{XY} & M^{12}_{XY} & 0 \\ M^{21}_{XY} & M^{22}_{XY} & \lambda_5 \langle \overline{N}^c_{\overline{H}} \rangle \\ \lambda_3 \langle N^c_H \rangle & 0 & M_\chi \end{pmatrix} . \tag{39}
$$

The determinant of above mass matrix is

$$\mathrm{Det}[M_{XYQ\overline{Q}}] \; = \; \left(M_{XY}^{11} M_{XY}^{22} - M_{XY}^{12} M_{XY}^{21}\right) M_\chi \sim M_{GUT}^2 M_\chi \; , \tag{40}$$

where we assume that there is no fine-tuning. So, there are two pairs of vector-like particles (major components belong to Q_Φ and $Q_{\Phi'}$, and $\overline{Q}_\Phi$ and $\overline{Q}_{\Phi'}$) with vector-like masses around the GUT scale, and one pair of vector-like particles (major components belong to Q_χ and $\overline{Q}_{\overline{\chi}}$) with vector-like mass around M_χ.

(2) The SM singlet mixings. For Φ and Φ', we consider the $SU(5) \times U(1)_X$ singlets as given in Eq. (27), corresponding to $U(1)_X$ gauge field component. We denote the singlets in Φ and Φ' as S_Φ and $S_{\Phi'}$. After the flipped $SU(5)$ gauge symmetry breaking, we have the following mass terms in the superpotential for the SM singlets S_Φ, S, $S_{\Phi'}$, S', N^c_χ, and $\overline{N}^c_{\overline{\chi}}$

$$\begin{aligned} W \; \supset \; & \frac{1}{2} M_{SX}^{11} S_\Phi^2 + M_{SX}^{12} S_\Phi S_{\Phi'} + \frac{1}{2} M_{SX}^{22} S_{\Phi'}^2 + \lambda_3 \langle N_H^c \rangle \overline{N}^c_{\overline{\chi}} (S_\Phi + \lambda_4 S) \\ & + \lambda_5 \langle \overline{N}^c_{\overline{H}} \rangle (S_{\Phi'} + \lambda_6 S') N^c_\chi + M_\chi \overline{N}^c_{\overline{\chi}} N^c_\chi \; , \end{aligned} \tag{41}$$

where M_{SX}^{ij} are mass parameters around the GUT scale. The corresponding mass matrix for the basis $(S_\Phi, S, S_{\Phi'}, S', N^c_\chi, \overline{N}^c_{\overline{\chi}})$ are

$$M_{\rm singlets} \; = \; \frac{1}{2} \begin{pmatrix} M_{SX}^{11} & 0 & M_{SX}^{12} & 0 & 0 & \lambda_3 \langle N_H^c \rangle \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 \lambda_4 \langle N_H^c \rangle \\ M_{SX}^{12} & 0 & M_{SX}^{22} & 0 & \lambda_5 \langle \overline{N}^c_{\overline{H}} \rangle & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \lambda_6 \langle \overline{N}^c_{\overline{H}} \rangle & 0 \\ 0 & 0 & \lambda_5 \langle \overline{N}^c_{\overline{H}} \rangle & \lambda_5 \lambda_6 \langle \overline{N}^c_{\overline{H}} \rangle & 0 & M_\chi \\ \lambda_3 \langle N_H^c \rangle & \lambda_3 \lambda_4 \langle N_H^c \rangle & 0 & 0 & M_\chi & 0 \end{pmatrix} . \tag{42}$$

The determinant of above mass matrix is

$$\mathrm{Det}[M_{\rm singlets}] \; = \; \frac{1}{64} \lambda_3^2 \lambda_4^2 \lambda_5^2 \lambda_6^2 \left[M_{SX}^{11} M_{SX}^{22} - (M_{SX}^{12})^2 \right] (\langle \overline{N}^c_{\overline{H}} \rangle)^2 (\langle N_H^c \rangle)^2 \sim M_{GUT}^2 M_{23}^4 \; . \tag{43}$$

Thus, there are two SM singlets (major components from S_Φ and $S_{\Phi'}$) with masses around the GUT scale, and four SM singlets with masses around the scale M_{23}. By the way, these SM singlets do not contribute to the RGE running below the M_{23} scale.

(3) The SM doublet mixings. After the flipped $SU(5)$ gauge symmetry breaking, we have the following mass terms in the superpotential for the SM doublets H_u, H_d, L_Σ, $\overline{L}_{\overline{\Sigma}}$, L_χ, and $\overline{L}_{\overline{\chi}}$

$$W \supset -4\lambda_3 V_\Phi L_\Sigma \overline{L}_{\overline{\chi}} - 4\lambda_5 V_{\Phi'} L_\chi \overline{L}_{\overline{\Sigma}} + 2\lambda_1 \langle N_H^c \rangle L_\Sigma H_u + 2\lambda_2 \langle \overline{N^c_{\overline{H}}} \rangle H_d \overline{L}_{\overline{\Sigma}} + \mu H_d H_u + M_\chi L_\chi \overline{L}_{\overline{\chi}} ~. \tag{44}$$

The corresponding mass matrix for the basis $(H_d, L_\Sigma, L_\chi)^t$ versus $(H_u, \overline{L}_{\overline{\Sigma}}, \overline{L}_{\overline{\chi}})$ are the following

$$M_{doublets} = \begin{pmatrix} \mu & 2\lambda_2 \langle \overline{N^c_{\overline{H}}} \rangle & 0 \\ 2\lambda_1 \langle N_H^c \rangle & 0 & -4\lambda_3 V_\Phi \\ 0 & -4\lambda_5 V_{\Phi'} & M_\chi \end{pmatrix} ~. \tag{45}$$

The determinant of above mass matrix is

$$\mathrm{Det}[M_{\mathrm{doublets}}] = -16\lambda_3\lambda_5 \mu V_\Phi V_{\Phi'} - 4\lambda_1\lambda_2 M_\chi \langle \overline{N^c_{\overline{H}}} \rangle \langle N_H^c \rangle ~. \tag{46}$$

Note that $V_\Phi \sim V_{\Phi'} \sim M_{GUT}$ and $\langle \overline{N^c_{\overline{H}}} \rangle = \langle N_H^c \rangle \sim M_{23}$, we obtain that there are two pairs of vector-like particles (major components belong to L_Σ and L_χ, and $\overline{L}_{\overline{\Sigma}}$ and $\overline{L}_{\overline{\chi}}$) with vector-like masses around the GUT scale, and one pair of vector-like particles (major components belong to H_d and H_u) whose vector-like mass M_{LD} is

$$M_{LD} \simeq \frac{\mathrm{Det}[M_{\mathrm{doublets}}]}{16\lambda_3\lambda_5 V_\Phi V_{\Phi'}} \sim -\mu - \frac{M_{23}^2}{M_{GUT}^2} M_\chi ~. \tag{47}$$

Because we need one pair of the Higgs doublets with mass around TeV scale to break the electroweak gauge symmetry, we obtain that μ should be around the TeV scale, and M_χ has a upper bound for a concrete model with gauge coupling unification. For example, with $M_{23} = 3.66 \times 10^{16}$ GeV and $M_{GUT} = 4.8 \times 10^{18}$ GeV as in the first case in the next subsection for gauge coupling unification, we obtain that $M_\chi \leq 1.72 \times 10^7$ GeV. Moreover, we emphasize that even if $\mu = 0$, we can generate the corresponding effective μ_{eff} term for one pair of the light Higgs doublets from above discussions.

With fine-tuning, there are two ways that we can have one pair of light Higgs doublets and very large vector-like mass M_χ for $\overline{\chi}_{\overline{F}}$ and χ_F. One way is that we fine-tune the two terms in Eq. (46) so that $\mathrm{Det}[M_{\mathrm{doublets}}] \sim \mu_{eff} M_{GUT}^2$ where $\mu_{eff} \sim 1$ TeV. The other way is that we replace the term $M_\chi \overline{\chi}\chi$ in the superpotential in Eq. (23) by the following two terms

$$W \supset y_\chi \overline{\chi}(\Phi - 3\lambda_4 S)\chi + y'_\chi \overline{\chi}(\Phi' - 3\lambda_6 S')\chi \ , \tag{48}$$

where y_χ and y'_χ are small Yukawa couplings. Note that

$$\begin{aligned} \langle\Phi\rangle - 3\lambda_4\langle S\rangle &= \mathrm{diag}(4,4,4,0,4,4,4,0,4,4,4,8,0,0,0,4)\, V_\Phi \ , \\ \langle\Phi'\rangle - 3\lambda_6\langle S'\rangle &= \mathrm{diag}(4,4,4,0,4,4,4,0,4,4,4,8,0,0,0,4)\, V_{\Phi'} \ , \end{aligned} \tag{49}$$

we have

$$W \supset 4y_\chi V_\Phi \left(\overline{\chi}_{\overline{F}}\chi_F + 2\overline{\chi}_l \chi_{\overline{l}}\right) + 4y'_\chi V_{\Phi'} \left(\overline{\chi}_{\overline{F}}\chi_F + 2\overline{\chi}_l \chi_{\overline{l}}\right) \ . \tag{50}$$

Thus, we obtain that the two terms in the superpotential in Eq. (48) will give vector-like masses to $\overline{\chi}_{\overline{F}}$ and χ_F, and $\overline{\chi}_l$ and $\chi_{\overline{l}}$, while they will not give vector-like mass to $\overline{\chi}_f$ and $\chi_{\overline{f}}$. And then we do not have the last term $M_\chi L_\chi \overline{L}_{\overline{\chi}}$ in Eq. (44), and the $(3,3)$ entry in the mass matrix in Eq. (45) is zero, *i. e.*, there is no M_χ entry in Eq. (45). Therefore, the vector-like mass for $\overline{\chi}_{\overline{F}}$ and χ_F can be any value below the M_{23} scale. By the way, in the concrete model building, we just need one term in the superpotential in Eq. (48).

B. Gauge Coupling Unification

We will study the gauge coupling unification. First, let us consider the masses for the additional particles. As discussed in the above subsection, there is one pair of vector-like particles (major components belong to Q_χ and $\overline{Q}_{\overline{\chi}}$) with vector-like mass around M_χ. Also, the particles D^c_χ and $\overline{D}^c_{\overline{\chi}}$ have vector-like mass M_χ. For simplicity, we assume that the correponding vector-like masses for these particles are the same, and we denote their masses as M_V because in the fine-tuning case, we may not have the $M_\chi \overline{\chi}\chi$ term in the superpotential in Eq. (23). We also assume that the masses for the color triplets of $h_{\mathbf{10}}$, H, $\overline{H}$, N^c_χ, and $\overline{N}^c_{\overline{\chi}}$ are around the $SU(2)_L \times SU(3)_C$ unification scale M_{23}, and the masses for the $\Sigma_{\bar{f}}$, $\Sigma_{\bar{l}}$, $\overline{\Sigma}_f$, $\overline{\Sigma}_l$, $\chi_{\bar{f}}$, $\chi_{\bar{l}}$, $\overline{\chi}_f$, $\overline{\chi}_l$, Φ, and Φ' are around the GUT scale M_{GUT}, where we do not write the particles in terms of mass eigenstates here. Moreover, we denote the Z-boson mass as M_Z, and the supersymmetry breaking scale as M_S. Also, the order of mass scales are assumed to be $M_Z \le M_S \le M_V \le M_{23} \le M_{GUT}$.

For gauge coupling unification, we consider the one-loop renormalizaton group equation (RGE) running for the gauge couplings because the two-loop effects only give minor corrections as long as the theory is perturbative. The generic one-loop RGEs for gauge couplings are

$$(4\pi)^2 \frac{d}{dt}\, g_i \;=\; b_i g_i^3 \;, \tag{51}$$

where $t = \ln \mu$ with μ being the renormalization scale, $g_1^2 \equiv 5g_Y^2/3$, and the g_Y, g_2, and g_3 are the gauge couplings for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ gauge groups, respectively.

The gauge coupling unification for the flipped $SU(5)$ is realized by first unifying α_2 and α_3 at scale M_{23}, then the gauge couplings of $SU(5)$ and $U(1)_X$ further unify at the scale M_{GUT}. From M_Z to M_S, the beta functions are $b^0 \equiv (b_1, b_2, b_3) = (41/10, -19/6, -7)$, and from M_S to M_V, the beta functions are $b^I = (33/5, 1, -3)$. From M_V to the α_2 and α_3 unification scale M_{23}, the beta functions are $b^{II} = (36/5, 4, 0)$.

Unification of α_2 and α_3 at the scale M_{23} gives the condition

$$\begin{aligned} \alpha_2^{-1}(M_Z) - \alpha_3^{-1}(M_Z) = \;\; & \frac{b_2^0 - b_3^0}{2\pi} \log\left(\frac{M_{SUSY}}{M_Z}\right) + \frac{b_2^I - b_3^I}{2\pi} \log\left(\frac{M_V}{M_{SUSY}}\right) \\ & + \frac{b_2^{II} - b_3^{II}}{2\pi} \log\left(\frac{M_{23}}{M_V}\right) \;, \end{aligned} \tag{52}$$

which can be solved to obtain the scale M_{23}.

The coupling α_1' of $U(1)_X$ is related to α_1 and α_5 at the scale M_{23} by

$$\alpha_1'^{-1}(M_{23}) = \frac{25}{24}\alpha_1^{-1}(M_{23}) - \frac{1}{24}\alpha_5^{-1}(M_{23}) \;. \tag{53}$$

And above the scale M_{23}, the beta functions for $U(1)_X$ and $SU(5)$ are $b^{III} \equiv (b_1', b_5) = (8, -2)$.

In our numerical calculations, we choose the central values of the strong coupling constant $\alpha_3(M_Z) = 0.1182 \pm 0.0027$ [19], and the fine structure constant α_{EM}, and weak mixing angle θ_W at M_Z to be [20]

$$\alpha_{EM}^{-1}(M_Z) = 128.91 \pm 0.02 \;, \quad \sin^2\theta_W(M_Z) = 0.23120 \pm 0.00015 \,. \tag{54}$$

Because the top quark pole mass is 172.7 ± 2.9 GeV [21], we might need supersymmetry breaking scale around or above the TeV scale to generate the large enough mass for the lightest CP even Higgs boson in the MSSM. So, we assume that $M_S = 10^3$ GeV. With

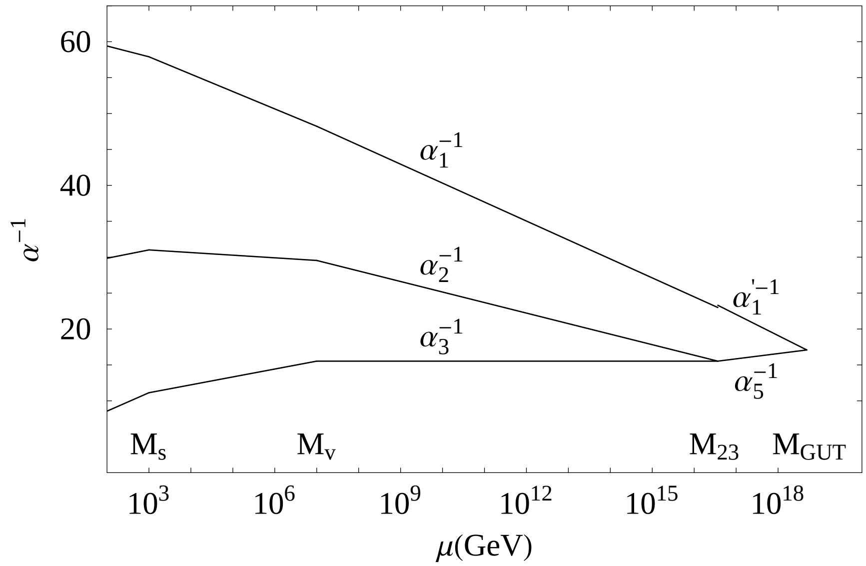

FIG. 1: The one-loop gauge coupling unification for $M_S = 10^3$ GeV and $M_V = 10^7$ GeV.

$M_V = 10^7$ GeV, we plot the gauge coupling unification in Fig. 1. We obtain that $M_{23} = 3.66 \times 10^{16}$ GeV, and $M_{GUT} = 4.8 \times 10^{18}$ GeV. Note that $M_{23}^2 M_V / M_{GUT} < 10^3$ GeV, we can have one pair of light Higgs doublets without any fine-tuning.

Since the GUT scale is close to the Planck scale 1.2×10^{19} GeV, we may need to include the one-loop supergravity contributions to the RGE running. It is reasonable to assume that similar to the non-supersymmetric gravity theory [22], the supergravity contributions to the one-loop RGEs of gauge couplings are still proportional to the gauge couplings linearly with the same coefficients for all the gauge couplings because the gravitons and gravitinos do not carry any gauge charge. Note that the gauge coupling of $U(1)_X$ is just a little bit smaller than that of $SU(5)$ at the renormalization scale close to the GUT scale, the supergravity contributions will only slightly increase the GUT scale [22].

As discussed in the above subsection, with fine-tuning we can have very large M_V. Assuming $M_S = 10^3$ GeV, we plot the GUT scale M_{GUT} versus M_V for M_V from 10^3 GeV to 10^{16} GeV in Fig. 2. Varying M_V will not change the scale M_{23} because these vector-like particles contribute the same one-loop beta functions to $SU(2)_L$ and $SU(3)_C$. Generically speaking, increasing M_V will decrease the GUT scale. In addition to the threshold corrections at the supersymmetry breaking scale due to the mass differences of the sparticles, it is well-known that there exist a few percent threshold corrections at the GUT scale in the concrete GUT models. So, the gauge coupling unification for M_V close to 10^6 GeV is still

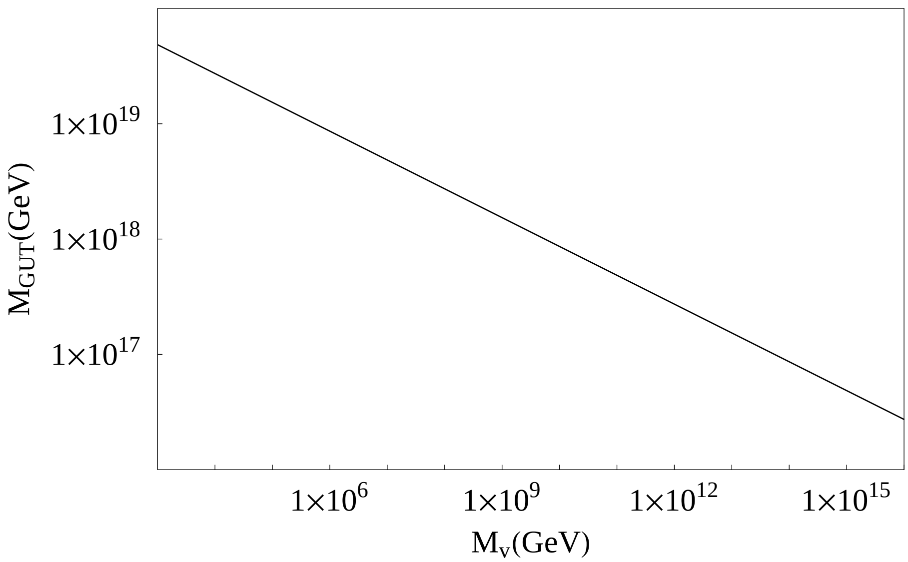

FIG. 2: The GUT scale M_{GUT} versus M_V for $M_S = 10^3$ GeV, and M_V from 10^3 GeV to 10^{16} GeV.

fine although there exists less than one percent discrepancy between the gauge couplings α_i^{-1}. It is interesting to have the GUT scale M_{GUT} around the string scale from 10^{17} GeV to 10^{18} GeV, and we find that the corresponding M_V scale is from 5.54×10^{13} GeV to 5.54×10^9 GeV.

High-scale supersymmetry breaking [23, 24, 25] is interesting due to the appearance of the string landscape [26] where we may explain the cosmological constant problem and gauge hierarchy problem [27, 28], and all the problems related to the low energy supersymmetry will be solved automatically if the supersymmetry breaking scale is higher than the PeV (10^{15} eV $\equiv 10^6$ GeV) scale [29]. Assuming $M_S = 10^6$ GeV and $M_V = 3 \times 10^8$ GeV, we plot the gauge coupling unification in Fig. 3. We obtain that $M_{23} = 4.88 \times 10^{16}$ GeV, and $M_{GUT} = 7.57 \times 10^{17}$ GeV. Note that $M_{23}^2 M_V / M_{GUT} \sim 1.25 \times 10^6$ GeV, we can also have one pair of light Higgs doublets at the PeV scale without fine-tuning. By the way, the SM Higgs doublet with electroweak-scale mass is obtained by fine-tuning the mass matrix for the scalar Higgs doublets.

V. REMARKS

We would like to briefly discuss the simplest $SO(10)$ model with flipped $SU(5)$ embedding where there is only one adjoint Higgs field, and we point out its major phenomenological

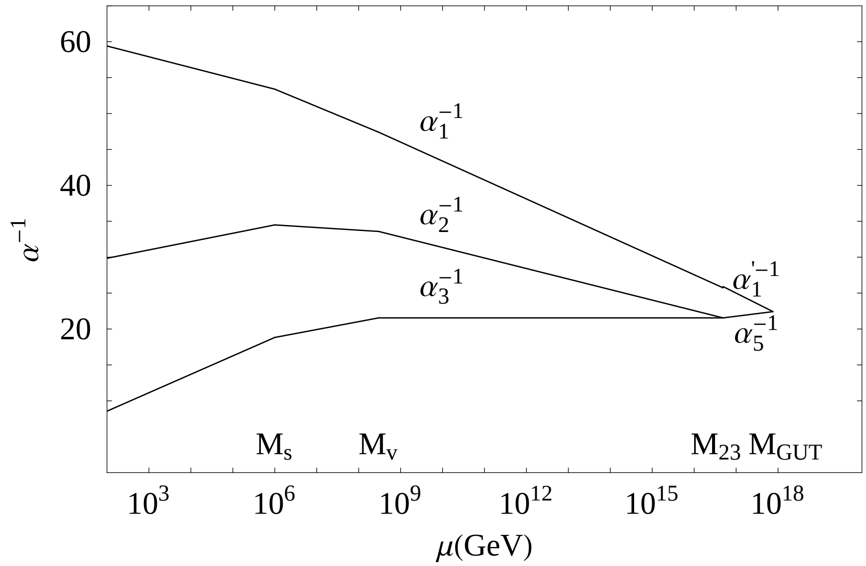

FIG. 3: The one-loop gauge coupling unification for $M_S = 10^6$ GeV and $M_V = 3 \times 10^8$ GeV.

difficulty. We will also explain how to generate the small mass for M_χ.

A. $SO(10)$ Model with One Adjoint Higgs Field

We can embed the flipped $SU(5)$ models into the $SO(10)$ model with only one adjoint Higgs field Φ. In the superpotential in Eq. (23), we change $W(\Phi, \Phi')$ to $W(\Phi)$, and replace the $\lambda_5 \overline{\Sigma}(\Phi' + \lambda_6 S')\chi$ term by the following term

$$W \supset \lambda_5 \overline{\Sigma}(\Phi + \lambda_6 S')\chi \ . \tag{55}$$

The discussions for the splittings of the five/one-plets and ten-plets in the multiplets $\overline{\chi}$ and Σ, and $\overline{\Sigma}$ and χ, are the same as those in the Section III except that we replace Φ' by Φ, and $V_{\Phi'}$ by V_Φ.

Let us concentrate on the problem. The mass matrix for the basis $(\overline{Q}_\Phi, \overline{Q}_{\overline{\chi}})^t$ versus (Q_Φ, Q_χ), are the following

$$M_{XYQ\overline{Q}} = \begin{pmatrix} M_{XY}^{11} & \lambda_5 \langle \overline{N^c_{\overline{H}}} \rangle \\ \lambda_3 \langle N^c_H \rangle & M_\chi \end{pmatrix} . \tag{56}$$

The determinant of above mass matrix is

$$\mathrm{Det}[M_{XYQ\overline{Q}}] = M_{XY}^{11} M_\chi - \lambda_3 \lambda_5 \langle \overline{N^c_{\overline{H}}} \rangle \langle N^c_H \rangle \ . \tag{57}$$

The discussions for the mass matrix of SM doublets are the same as those in the subsection A in Section IV except that we change $V_{\Phi'}$ to V_{Φ} in Eqs. (44) and (45). So, without fine-tuning the M_{χ} still cannot be larger than about 10^8 GeV. Then we have

$$\mathrm{Det}[M_{XYQ\overline{Q}}] \sim -\lambda_3\lambda_5\langle \overline{N}^c_{\overline{H}}\rangle\langle N^c_H\rangle \sim -M_{23}^2 \ . \tag{58}$$

Thus, there is one pair of vector-like particles (major components belong to Q_{Φ} and $\overline{Q}_{\Phi}$) with vector-like mass around the GUT scale, and one pair of vector-like particles (major components belong to Q_{χ} and $\overline{Q}_{\overline{\chi}}$) with vector-like mass around M_{23}^2/M_{GUT}. Note that the particles D^c_{χ} and $\overline{D}^c_{\overline{\chi}}$ have vector-like mass M_{χ}, we can easily show that the gauge coupling unification can not be realized. By the way, with large fine-tuning so that M_{χ} can be around M_{23}^2/M_{GUT} and $\mathrm{Det}[M_{XYQ\overline{Q}}] \sim 10^{-2}M_{23}^2$, we can have gauge coupling unification.

With $M_{\chi} \leq 10^8$ GeV and without fine-tuning, we may also achieve the gauge coupling unification by adding extra vector-like particles, for example, one or two pairs of $\overline{\mathbf{16}}$ and $\mathbf{16}$. However, these models are very complicated in general, and still need some fine-tuning to achieve the gauge coupling unification after detailed study.

B. Explanation to the Suitable Mass M_{χ}

To have the natural models, we need to explain why M_{χ} can be around 10^7 GeV. There are two well-known ways to generate small masses: the Froggat-Nielsen mechanism [30] and the see-saw mechanism [3]. Because we will try to generate the SM fermion masses and mixings, and the suitable mass M_{χ} via Froggat-Nielsen mechanism by introducing extra flavour symmetry in our models in a future publication, we employ the see-saw mechanism to explain the M_{χ} here.

As we know, an elegant and popular solution to the strong CP problem is the Peccei-Quinn mechanism [31], in which a global axial symmetry $U(1)_{PQ}$ is introduced and broken spontaneously at some high energy scale. The original Weinberg–Wilczek axion [32] is excluded by experiment, in particular by the non-observation of the rare decay $K \rightarrow \pi + a$ [33] where a is the axion field. There are two viable "invisible" axion models in which the experimental bounds can be evaded: (1) the Kim–Shifman–Vainshtein–Zakharov (KSVZ) axion model, which introduces a SM singlet S_{PQ} and a pair of extra vector-like quarks that carry $U(1)_{PQ}$ charges while the SM fermions and Higgs fields are neutral under $U(1)_{PQ}$

symmetry [34]; (2) the Dine–Fischler–Srednicki–Zhitnitskii (DFSZ) axion model, in which a SM singlet S_{PQ} and one pair of Higgs doublets are introduced, and the SM fermions and Higgs fields are also charged under $U(1)_{PQ}$ symmetry [35]. From laboratory, astrophysical, and cosmological constraints, the $U(1)_{PQ}$ symmetry breaking scale is limited to the range from 10^{10} GeV to 10^{12} GeV [33]. And then the VEV of S_{PQ} is also roughly in the range from 10^{10} GeV to 10^{12} GeV. Interestingly, $(\langle S_{PQ}\rangle)^2/M_{23}$ can be from 10^4 GeV to 10^8 GeV, which can give us the needed mass scale for M_χ.

Let us introduce one pair of the spinor $\overline{\mathbf{16}}$ and $\mathbf{16}$ representation vector-like particles $\overline{\chi}'$ and χ'. In the superpotential in Eq. (23), we can forbid the $M_\chi \overline{\chi}\chi$ term by $U(1)_{PQ}$ symmetry, and introduce the following superpotential

$$W \supset M_{\chi'}\overline{\chi}'\chi' + \lambda_{PQ1} S_{PQ}\overline{\chi}'\chi + \lambda_{PQ2} S_{PQ}\overline{\chi}\chi' ~, \tag{59}$$

where λ_{PQ1} and λ_{PQ2} are the Yukawa couplings, and $M_{\chi'}$ is a mass parameter around the scale M_{23} which can be generated via Froggat-Nielsen mechanism easily.

Because we are not interested in the superheavy states that are always superheavy without fine-tuning, let us focus on the mixings between the light states $\overline{\chi}_{\overline{F}}$ and χ_F of $\overline{\chi}$ and χ and the superheavy states $\overline{\chi}'_{\overline{F}}$ and χ'_F of $\overline{\chi}'$ and χ'. After the $U(1)_{PQ}$ symmetry breaking, the mass matrix for the basis $(\overline{\chi}_{\overline{F}}, \overline{\chi}'_{\overline{F}})^t$ versus (χ_F, χ'_F) is

$$M_{\chi_F\chi'_F} = \begin{pmatrix} 0 & \lambda_{PQ2}\langle S_{PQ}\rangle \\ \lambda_{PQ1}\langle S_{PQ}\rangle & M_{\chi'} \end{pmatrix} . \tag{60}$$

Thus, we obtain that there is one pair of vector-like particles (major components belong to $\overline{\chi}'_{\overline{F}}$ and χ'_F) with vector-like mass around the GUT scale, and one pair of vector-like particles (major components belong to χ_F and $\overline{\chi}_{\overline{F}}$) with vector-like mass around

$$M_{\rm light~\chi_F} \sim \frac{\lambda_{PQ1}\lambda_{PQ2}(\langle S_{PQ}\rangle)^2}{M_{\chi'}} \sim 10^{4-8}~{\rm GeV} ~. \tag{61}$$

In fact, we can simply integrate out vector-like particles $\overline{\chi}'$ and χ' in Eq. (59), and obtain the following superpotential

$$W \supset -\lambda_{PQ1}\lambda_{PQ2}\frac{S_{PQ}^2}{M_{\chi'}}\overline{\chi}\chi ~. \tag{62}$$

This is the exact high-dimensional operator that can generate the suitable vector-like mass M_χ. In short, we can indeed generate the light M_χ naturally.

VI. DISCUSSION AND CONCLUSIONS

We embedded the flipped $SU(5)$ models into the $SO(10)$ models. After the $SO(10)$ gauge symmetry is broken down to the flipped $SU(5)$ gauge symmetry, we can split the five/one-plets and ten-plets in the multiplets $\overline{\chi}$ and Σ, and $\overline{\Sigma}$ and χ via the stable sliding singlet mechanism. Similar to the flipped $SU(5)$ model, the gauge symmetry can be broken down to the SM gauge symmetry by giving VEVs to the singlet componets of H and $\overline{H}$. The doublet-triplet splitting problem can be solved naturally by the missing partner mechanism, and the Higgsino-exchange mediated proton decay can be avoided elegantly. Moreover, we showed that there exists one pair of the light Higgs doublets with major components from H_u and H_d for the electroweak gauge symmetry breaking. Because there exist two pairs of the vector-like fields with similar intermediate-scale masses (major components from Q_χ and $\overline{Q}_{\overline{\chi}}$, and D^c_χ and $\overline{D}^c_{\overline{\chi}}$), we can have gauge coupling unification at the GUT scale which is reasonably (about one or two orders) higher than the $SU(2)_L \times SU(3)_C$ unification scale. In short, we can keep the beautiful features and get rid of the drawbacks of the flipped $SU(5)$ models in our $SO(10)$ models.

Furthermore, we briefly studied the simplest $SO(10)$ model with flipped $SU(5)$ embedding, and found that it can not work without fine-tuning. We also explained how to generate the suitable vector-like mass M_χ for χ and $\overline{\chi}$.

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APPENDIX A: THE $SO(10)$ GENERATORS IN THE SPINOR REPRESENTATIONS

The $SO(10)$ generators in the spinor representations and the assignment of the SM fermions in the **16** can be found in Ref. [36]. We copy the $\sigma \cdot W_\mu$, and rename it as $\not{A}_\mu$. The

16×16 matrix for $\not{A}_\mu$ can be re-written into the following four 8×8 matrices

$$\not{A} = \begin{pmatrix} \not{A}_{11} & \not{A}_{12} \\ \not{A}_{21} & \not{A}_{22} \end{pmatrix} , \tag{A1}$$

with

$$\not{A}_{11} = \begin{pmatrix} \lambda_{11} & V_{12} & V_{13} & X_1^0 & W_L^- & & & \\ V_{12}^* & \lambda_{22} & V_{23} & X_2^- & & W_L^- & & \\ V_{13}^* & V_{23}^* & \lambda_{33} & X_3^- & & & W_L^- & \\ \overline{X}_1^0 & X_2^+ & X_3^+ & \lambda_{44} & & & & W_L^- \\ W_L^+ & & & & \lambda_{55} & V_{12} & V_{13} & X_1^0 \\ & W_L^+ & & & V_{12}^* & \lambda_{66} & V_{23} & X_2^- \\ & & W_L^+ & & V_{13}^* & V_{23}^* & \lambda_{77} & X_3^- \\ & & & W_L^+ & \overline{X}_1^0 & X_2^+ & X_3^+ & \lambda_{88} \end{pmatrix} ,$$

$$\not{A}_{12} = \begin{pmatrix} 0 & A_6^0 & -A_5^0 & -Y_1^+ & 0 & -Y_6^- & Y_5^- & -\overline{A}_1^0 \\ -A_6^0 & 0 & A_4^- & -\overline{Y}_2^0 & Y_6^- & 0 & -Y_4^{--} & -A_2^- \\ A_5^0 & -A_4^- & 0 & -\overline{Y}_3^0 & -Y_5^- & Y_4^{--} & 0 & -A_3^- \\ Y_1^+ & \overline{Y}_2^0 & \overline{Y}_3^0 & 0 & \overline{A}_1^0 & A_2^- & A_3^- & 0 \\ 0 & -A_3^+ & A_2^+ & -Y_4^{++} & 0 & Y_3^0 & -Y_2^0 & -A_4^+ \\ A_3^+ & 0 & A_1^0 & -Y_5^+ & Y_3^0 & 0 & -Y_1^- & -\overline{A}_5^0 \\ -A_2^+ & A_1^0 & 0 & -Y_6^+ & Y_2^0 & -Y_1^- & 0 & -\overline{A}_6^0 \\ Y_4^{++} & Y_5^+ & Y_6^+ & 0 & A_4^+ & \overline{A}_5^0 & \overline{A}_6^0 & 0 \end{pmatrix} ,$$

$$
\not{A}_{21} = \begin{pmatrix}
0 & -\overline{A}_6^0 & \overline{A}_5^0 & Y_1^- & 0 & A_3^- & -A_2^- & Y_4^{--} \\
\overline{A}_6^0 & 0 & -A_4^+ & Y_2^0 & -A_3^- & 0 & \overline{A}_1^0 & Y_5^- \\
-\overline{A}_5^0 & A_4^+ & 0 & Y_3^0 & A_2^- & -\overline{A}_1^0 & 0 & Y_6^- \\
-Y_1^- & -Y_2^0 & -Y_3^0 & 0 & -Y_4^- & -Y_5^- & -Y_6^- & 0 \\
0 & Y_6^+ & -Y_5^+ & A_1^0 & 0 & -\overline{Y}_3^0 & \overline{Y}_2^0 & A_4^- \\
-Y_6^+ & 0 & Y_4^{++} & A_2^+ & \overline{Y}_3^0 & 0 & -Y_1^+ & A_5^0 \\
Y_5^+ & -Y_4^{++} & 0 & A_3^+ & -\overline{Y}_2^0 & Y_1^+ & 0 & A_6^0 \\
-A_1^0 & -A_2^+ & -A_3^+ & 0 & -A_4^- & -A_5^0 & -A_6^0 & 0
\end{pmatrix} ,
$$

$$
\not{A}_{22} = \begin{pmatrix}
\lambda_{99} & -V_{12}^* & -V_{13}^* & -\overline{X}_1^0 & W_R^- & & & \\
-V_{12} & \lambda_{1010} & -V_{23}^* & -X_2^+ & & W_R^- & & \\
-V_{13} & -V_{23} & \lambda_{1111} & -X_3^+ & & & W_R^- & \\
-X_1^0 & -X_2^- & -X_3^- & \lambda_{1212} & & & & W_R^- \\
W_R^+ & & & & \lambda_{1313} & -V_{12}^* & -V_{13}^* & -\overline{X}_1^0 \\
 & W_R^+ & & & -V_{12} & \lambda_{1414} & -V_{23}^* & -X_2^+ \\
 & & W_R^+ & & -V_{13} & -V_{23} & \lambda_{1515} & -X_3^+ \\
 & & & W_R^+ & -X_1^0 & -X_2^- & -X_3^3 & \lambda_{1616}
\end{pmatrix} .
$$

The 45 gauge bosons consist of 12 A, 6 X, 6 V, 12 Y, 2 charged W_L, 2 charged W_R, and 16 λ which can be rewritten as 5 independent fields, V_3, V_8, V_{15}, W_L^0 and W_R^0.

The first family of the SM fermions forms a spinor **16** representation

$$
\mathbf{16}_1 = (u_r, u_g, u_b, \nu_e, d_r, d_g, d_b, e^-, d_r^c, d_g^c, d_b^c, e^+, -u_r^c, -u_g^c, -u_b^c, -\nu_e^c)^t \ , \tag{A2}
$$

similarly for the second and third families. As the $SO(10)$ is broken down to $SU(5) \times U(1)$ or flipped $SU(5)$, the spinor representation **16** is decomposed as

$$
\mathbf{16} \to (\mathbf{10}, \mathbf{1}) + (\overline{\mathbf{5}}, -\mathbf{3}) + (\mathbf{1}, \mathbf{5}) \ , \tag{A3}
$$

where

$$(\mathbf{10},\mathbf{1}) = (Q, U^c, E^c), \quad (\overline{\mathbf{5}}, -\mathbf{3}) = (D^c, L), \quad \text{and} \quad (\mathbf{1},\mathbf{5}) = N^c \tag{A4}$$

for breaking to $SU(5) \times U(1)$, and

$$(\mathbf{10},\mathbf{1}) = (Q, D^c, N^c), \quad (\overline{\mathbf{5}}, -\mathbf{3}) = (U^c, L), \quad \text{and} \quad (\mathbf{1},\mathbf{5}) = E^c \tag{A5}$$

for breaking to flipped $SU(5)$.

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